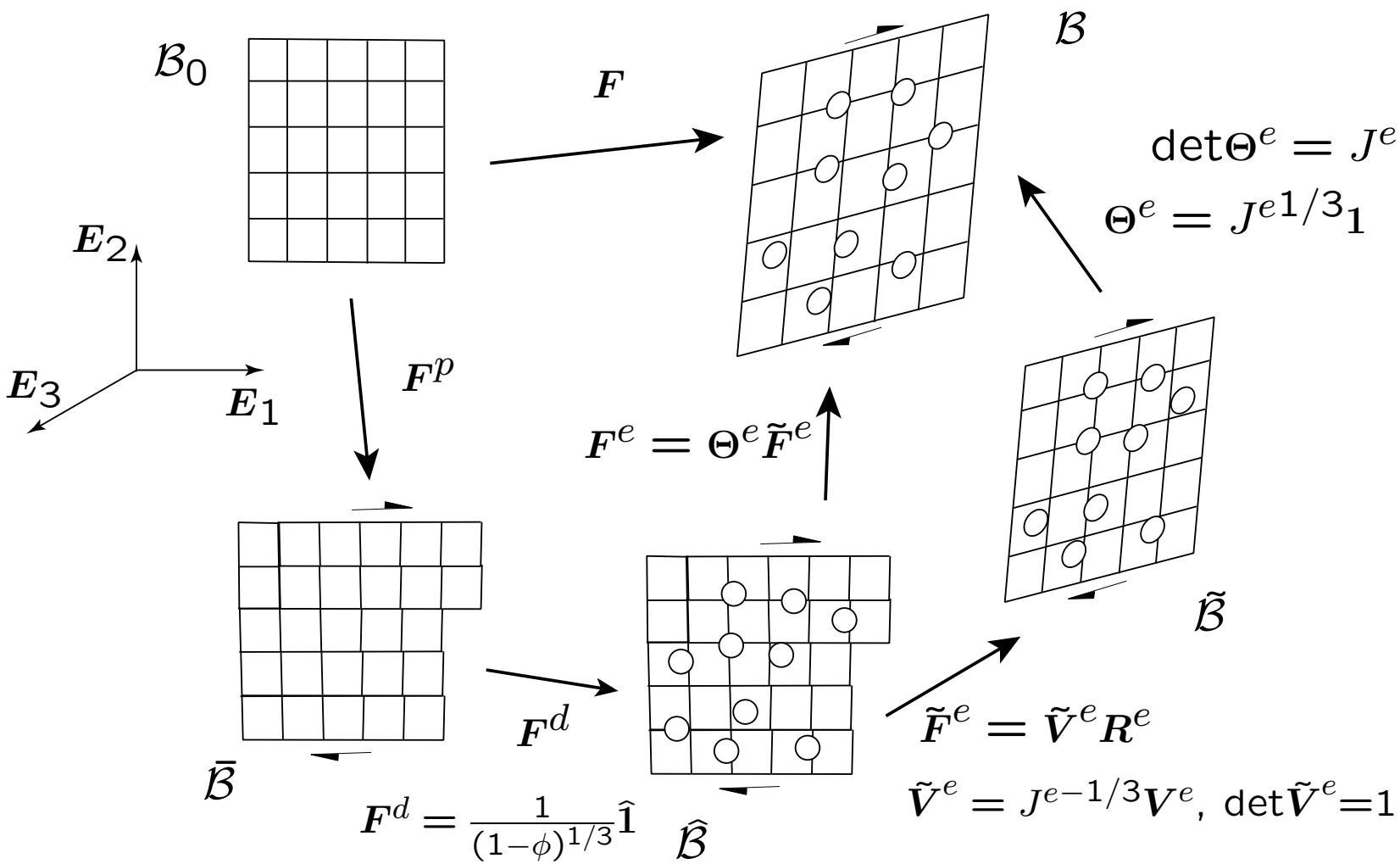
Kinematics

- **multiplicative decomposition**: Bilby et al. (1955), Kroner (1960), Lee & Liu (1967), Lee (1969), Davison et al. (1977); vol./dev. split (Flory 1961, Simo et al. 1985); Regueiro 2002:

$$\boldsymbol{F} = \boldsymbol{F}^e\boldsymbol{F}^d\boldsymbol{F}^p = \boldsymbol{\Theta}^e\tilde{\boldsymbol{F}}^e\boldsymbol{F}^d\boldsymbol{F}^p\,, \quad F_{iI} = \Theta^e_{ij}\tilde{F}^e_{j\hat{I}}F^d_{\hat{I}\bar{I}}F^p_{\bar{I}I}\,, \quad J = \det\boldsymbol{F} = \frac{J^e}{1-\phi}$$

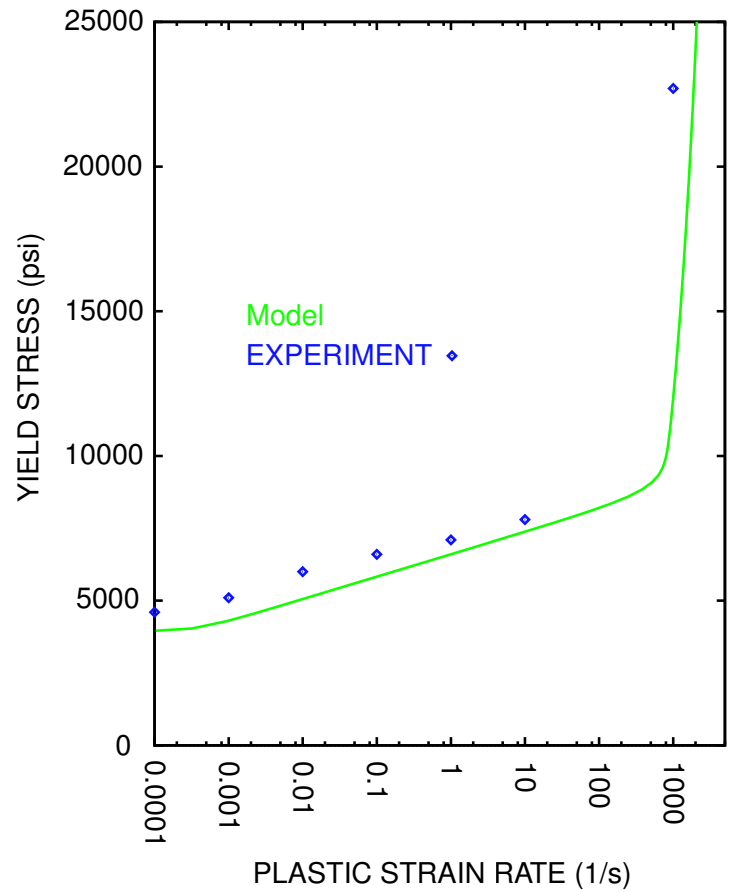


Flow Stress

(*thermally-activated dislocation motion*, Bammann et al. 1996; *dislocation drag*, Klahn et al. 1971, Teodosiu & Sidoroff 1976)

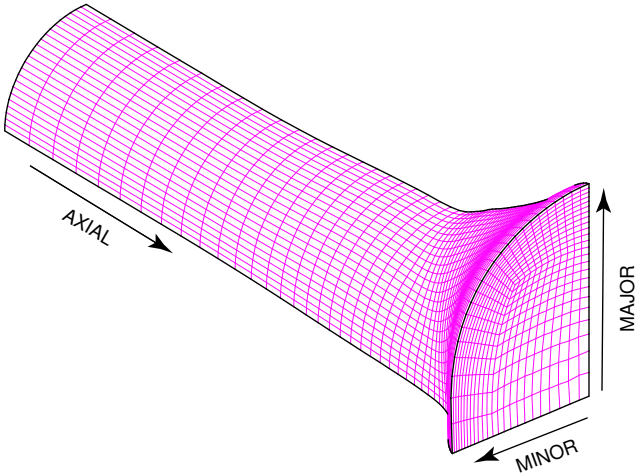
- **plastic flow kinetic equation**:

$$
\begin{aligned}
\dot{\epsilon}^{p,\text{eff}} &:= \frac{\dot{\epsilon}_1 \dot{\epsilon}_2}{\dot{\epsilon}_1 + \dot{\epsilon}_2} \\
\dot{\epsilon}_1 &= f(\theta) \left[\sinh \left(\frac{\widehat{S}^{\text{eff}}}{h_3 \widehat{\chi}(\widehat{\kappa} + Y(\theta))} - 1 \right) \right]^m \\
\dot{\epsilon}_2 &= \frac{1}{B} \left(\frac{\widehat{S}^{\text{eff}}}{h_3 \widehat{\chi}(\widehat{\kappa} + Y(\theta))} - Y^B \right)
\end{aligned}
$$



Texture effects

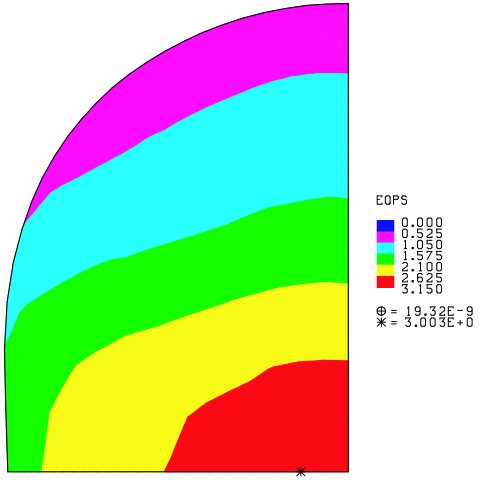
(Prantil et al. 1993, Bammann et al. 2001)



- **plastic spin**:

$$\widehat{\boldsymbol{W}}^p = \lambda_g \left(\widehat{\boldsymbol{A}} \cdot \widehat{\boldsymbol{D}}^p - \widehat{\boldsymbol{D}}^p \cdot \widehat{\boldsymbol{A}} \right)$$

- **structure tensor**: $\widehat{\boldsymbol{A}}$



Associative vs. Nonassociative Inelastic Flow

- **inelastic flow rule** (Marin & McDowell 1996):

$$\begin{aligned}\widehat{\boldsymbol{D}}^i &:= \widehat{\boldsymbol{D}}^d + \widehat{\boldsymbol{D}}^p = \frac{\dot{\phi}}{3(1-\phi)}\widehat{\mathbf{1}} + \operatorname{sym}\left(\widetilde{\boldsymbol{C}}^e \widehat{\boldsymbol{L}}^p\right) \\ &:= \frac{\widehat{G}(\operatorname{dev}\widehat{\boldsymbol{S}}, \widehat{p}, \phi, m)}{\sqrt{3}} \|\widehat{\boldsymbol{D}}^p\| \, \widehat{\mathbf{1}} + \|\widehat{\boldsymbol{D}}^p\| \, \widehat{\boldsymbol{N}}^p \\ &= \underbrace{\|\widehat{\boldsymbol{D}}^p\| \sqrt{1+\widehat{G}^2}}_{\|\widehat{\boldsymbol{D}}^i\|} \underbrace{\frac{1}{\sqrt{1+\widehat{G}^2}} \left(\widehat{\boldsymbol{N}}^p + \frac{\widehat{G}}{\sqrt{3}}\widehat{\mathbf{1}}\right)}_{\widehat{\boldsymbol{N}}^i}\end{aligned}$$

- **isotropic void growth**:

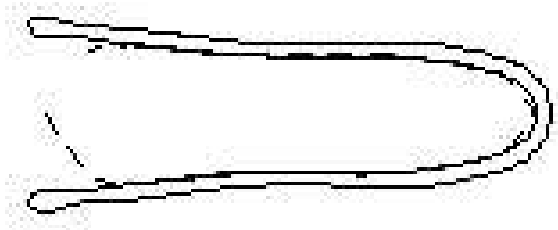
$$\dot{\phi} = \frac{3(1-\phi)}{\sqrt{3}} \widehat{G}(\operatorname{dev}\widehat{\boldsymbol{S}}, \widehat{p}, \phi, m) \, \|\widehat{\boldsymbol{D}}^p\|$$

- **associative**:

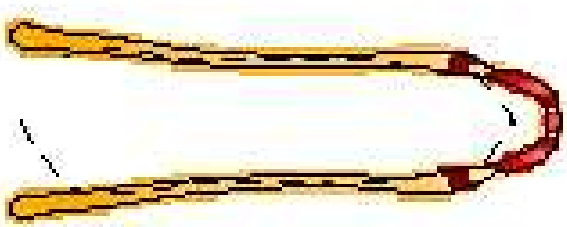
$$\widehat{\boldsymbol{N}}^\sigma : \widehat{\boldsymbol{N}}^i = 1 \quad \rightarrow \quad \widehat{G}$$

- **nonassociative**: choose $\widehat{G}$ such that

$$\widehat{\boldsymbol{N}}^\sigma : \widehat{\boldsymbol{N}}^i \neq 1$$



associative, pressure dependent,
Cocks 1989



non-associative, pressure dependent,

Cocks & Ashby 1980

non-associative, pressure independent,

Cocks & Ashby 1980